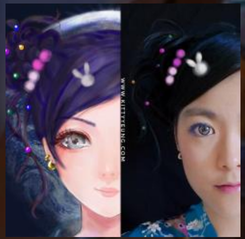
Introduction to Quantum Computing



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@artbyphysicistkittyyeung

June 21, 2020
Hackaday, session 12
Other communities, session 4

HackadayU

REVERSE ENGINEERING
Learn the fundamentals of reverse engineering using Ghidra
SUMMER 2020

KICAD & FREECAD 101
Learn the basics of circuit design with open source tools
SUMMER 2020

QUANTUM COMPUTING
Learn the fundamentals of Quantum Computing
SUMMER 2020

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an alternative grad school for hardware hackers

Learn from experienced instructors from all over the world. Available for free to everyone. All proceeds go towards socially conscious causes.

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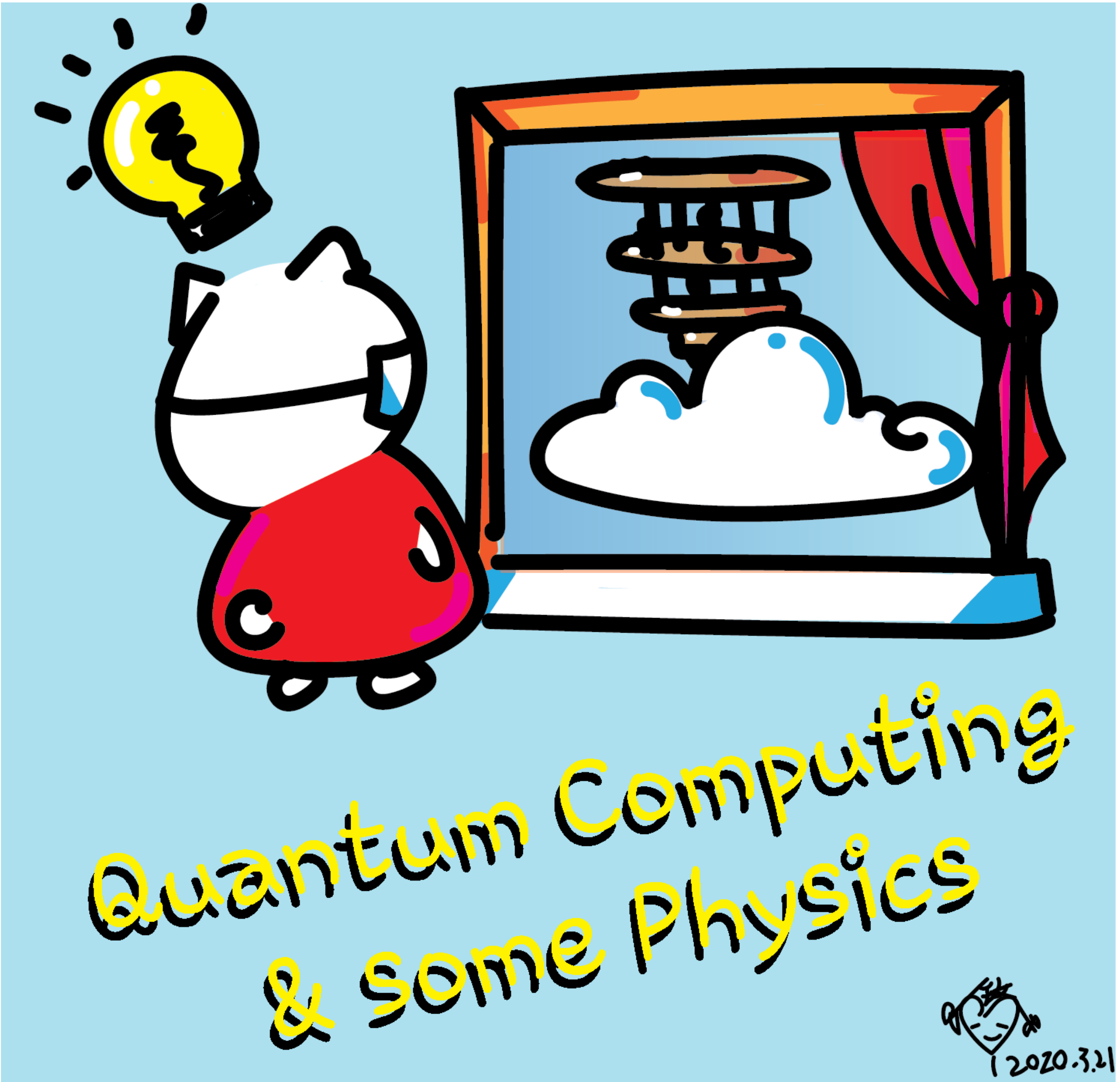
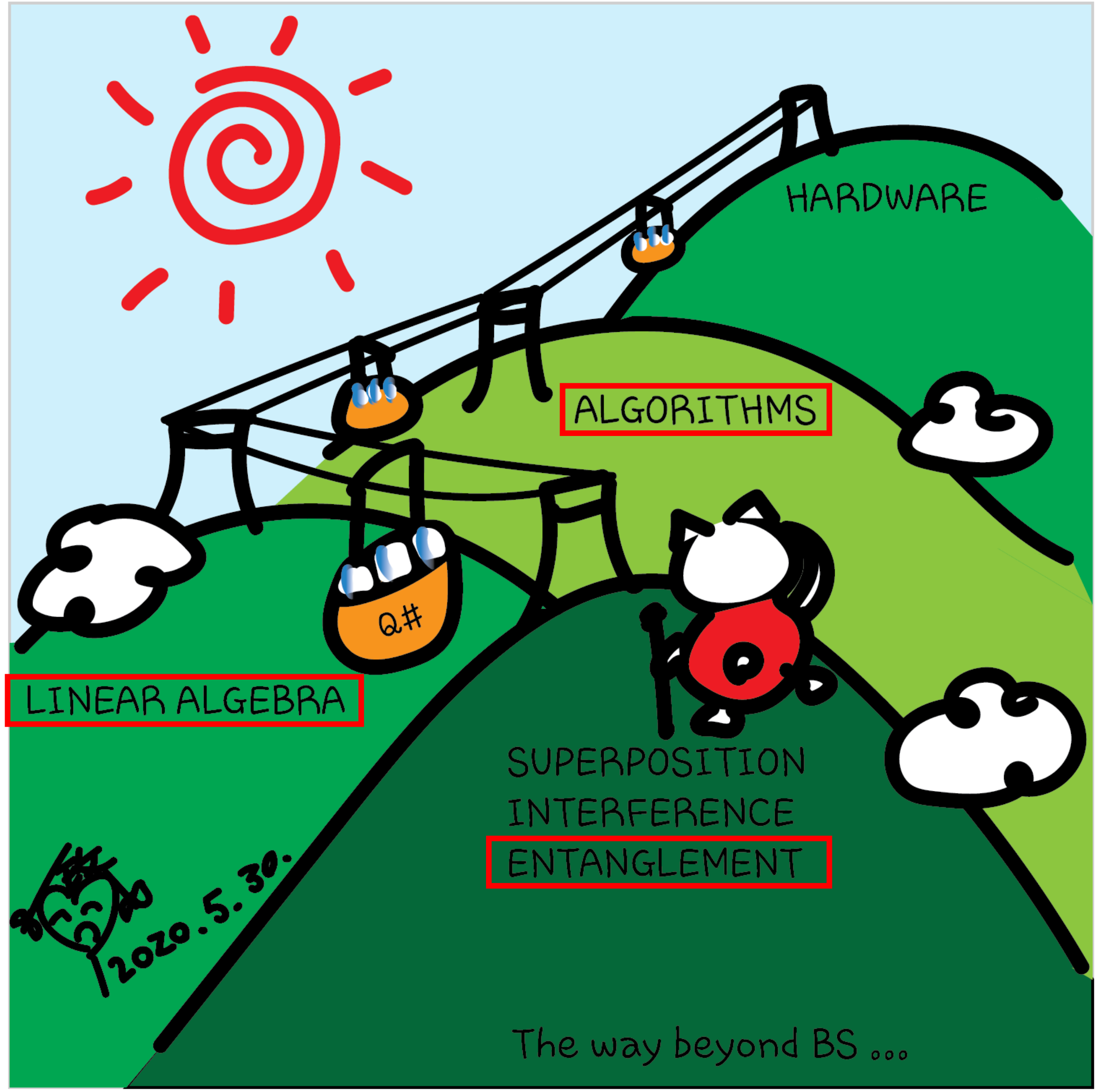
Class structure

- Comics on Hackaday – Introduction to Quantum Computing every Sun
- 30 mins – 1 hour every Sun, one concept (theory, hardware, programming), Q&A
- Contribute to Q# documentation http://docs.microsoft.com/quantum
- Coding through Quantum Katas https://github.com/Microsoft/QuantumKatas/
- Discuss in Hackaday project comments throughout the week
- Take notes

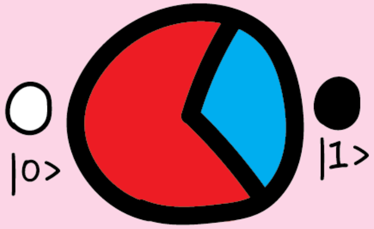



HARDWARE
ALGORITHMS
Q#
LINEAR ALGEBRA
SUPERPOSITION
INTERFERENCE
ENTANGLEMENT
2020.5.30.
The way beyond BS ...

2020.3.28.

A qubit system is all the possible configurations in superposition.

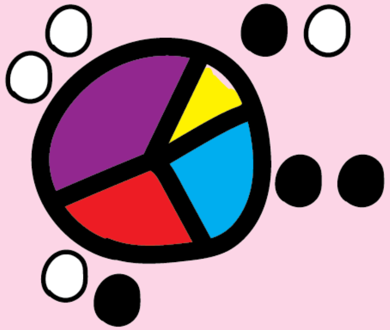
PIE CHART DENOTING PROBABILITY OF EACH CONFIGURATION



ONE QUBIT, TWO CONFIGURATIONS:

$a|0\rangle+b|1\rangle$

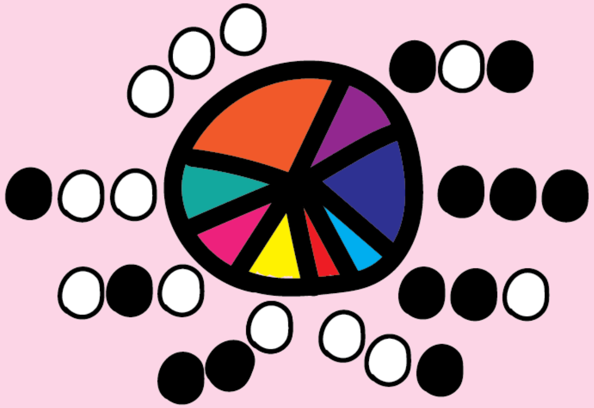
$a^2+b^2=1$ (total probability adds up to 1)



TWO QUBITS, FOUR CONFIGURATIONS:

$a|00\rangle+b|01\rangle+c|10\rangle+d|11\rangle$

$a^2+b^2+c^2+d^2=1$



...

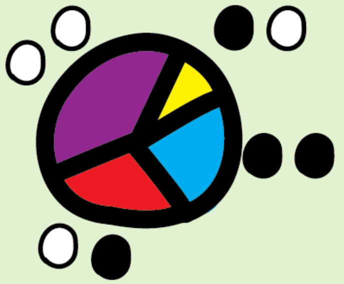
N qubits will have 2^N possible configurations in superposition!

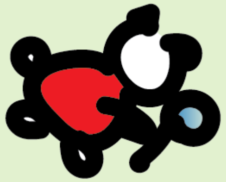
THREE QUBITS, EIGHT CONFIGURATIONS:

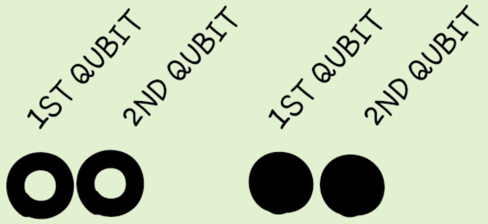
$a|000\rangle+b|001\rangle+c|010\rangle+d|100\rangle+e|110\rangle+f|101\rangle+g|011\rangle+h|111\rangle$

$a^2+b^2+c^2+d^2+e^2+f^2+g^2+h^2=1$

We've seen in page 9 that with two qubits, there are four possible configurations: both qubits in |0>s or |1>s, or one in |0> with the other in |1>. What if we make the |0>|0> case in superposition with the |1>|1> case? Or |0>|1> in superposition with |1>|0>?



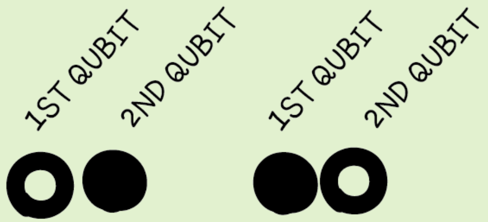




$a|0 \quad 0\rangle + d|1 \quad 1\rangle$

If we set the system to be in this case, we know that if we measure the first qubit and get |0>, the second qubit must be in |0>, without needing to measure it.

We can also measure the second qubit to know what the first qubit is without measuring it.



$b|0 \quad 1\rangle + c|1 \quad 0\rangle$

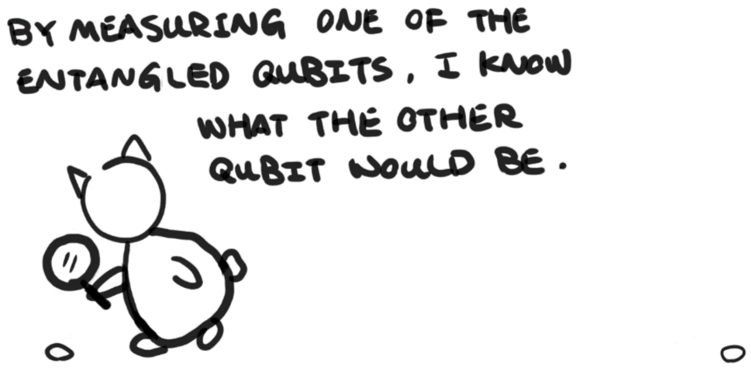
Similarly in this case, if the first qubit is |0>, the second qubit must be |1>. If the first is |1>, the second must be |0>.

The qubits are correlated. This is called "**entanglement**".

Entanglement

Bell states

$|\varphi^{\pm}\rangle = \frac{|01\rangle \pm |10\rangle}{\sqrt{2}}$ and $|\phi^{\pm}\rangle = \frac{|00\rangle \pm |11\rangle}{\sqrt{2}}$



Take $|\phi^{+}\rangle$ as an example, upon measuring the first qubit, one obtains two possible results:

1. First qubit is 0, get a state $|\phi'\rangle = |00\rangle$ with probability ½.
2. First qubit is 1, get a state $|\phi''\rangle = |11\rangle$ with probability ½.

If the second qubit is measured, the result is the same as the above. This means that measuring one qubit tells us what the other qubit is.

Entanglement

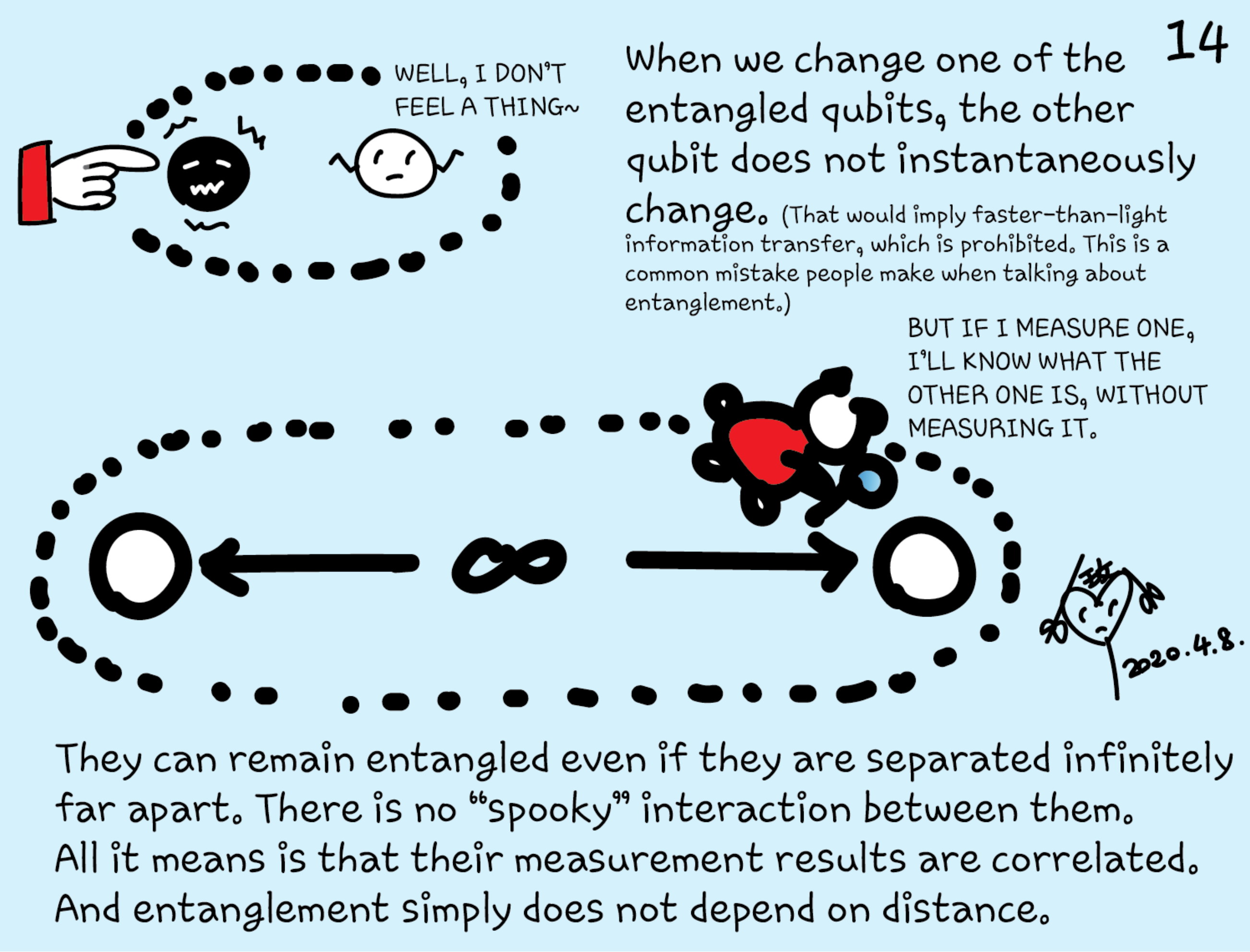
Math insert – entangled states cannot be factored back to individual qubits--------------

Remember in section 1.1, a two-qubit state can be obtained by doing a tensor product of two individual one-qubit states. However, a Bell state cannot be factored back into two individual qubits. For example,

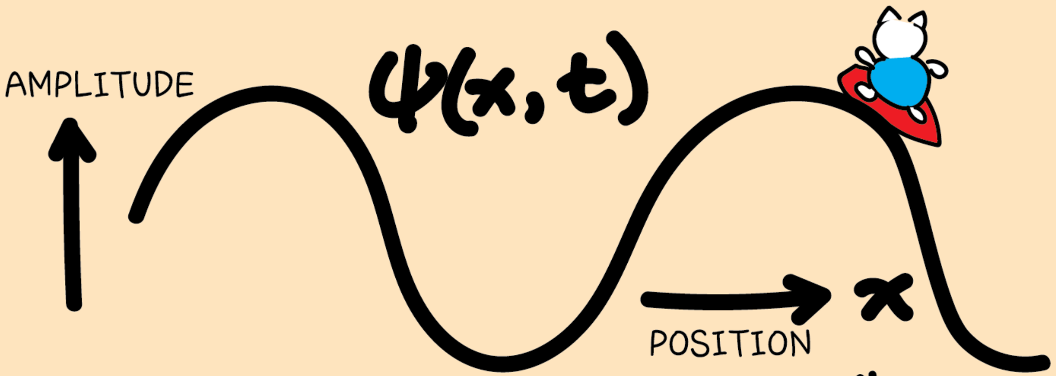
$$|\phi^{\pm}\rangle = \frac{|00\rangle \pm |11\rangle}{\sqrt{2}} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}.$$

If we want to factor it back to two separate qubits as in $\begin{pmatrix} a \\ b \end{pmatrix} \otimes \begin{pmatrix} c \\ d \end{pmatrix}$, then this set of equations need to be simultaneously satisfied

$ac = \frac{1}{\sqrt{2}}$, $ad = 0, bc = 0$ and $bd = \frac{1}{\sqrt{2}}$. Unfortunately, it is impossible. This set of equations has no solution. It can only be 50% chance of getting $|00\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ or $|11\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.



[Check out more commonly made mistakes](https://quantumfactsheet.github.io/)
https://quantumfactsheet.github.io/



Classical wave, e.g. mass on a spring, water wave, sound wave, pendulum, etc.

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}$$

Electromagnetic wave, derived from Maxwell's equations

$$\nabla^2 E = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 H = \mu_0 \varepsilon_0 \frac{\partial^2 H}{\partial t^2}$$

Quantum wave, Schrödinger equation

$$-\frac{\hbar^2}{2m}\nabla^2 \psi = i\hbar \frac{\partial \psi}{\partial t}$$

$$\left.\begin{aligned} \nabla \cdot E &= 0 \\ \nabla \cdot H &= 0 \\ \nabla \times E &= -\mu_0 \frac{\partial H}{\partial t} \\ \nabla \times H &= \varepsilon_0 \frac{\partial E}{\partial t} \end{aligned}\right\}$$

MAXWELL'S EQUATIONS FOR ELECTROMAGNETISM

REARRANGING

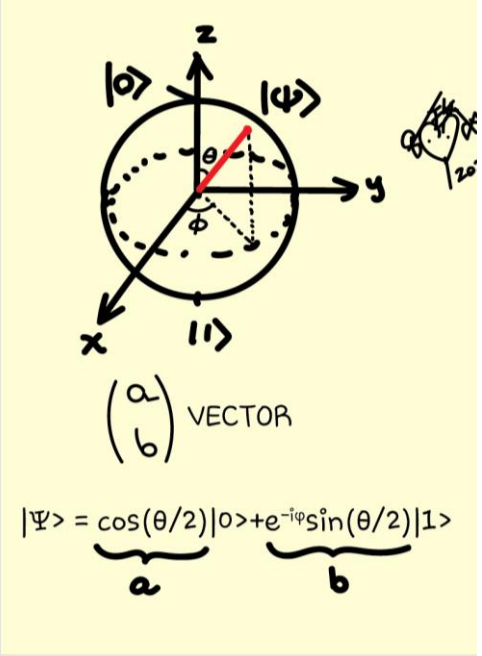
$$\nabla^2 E = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

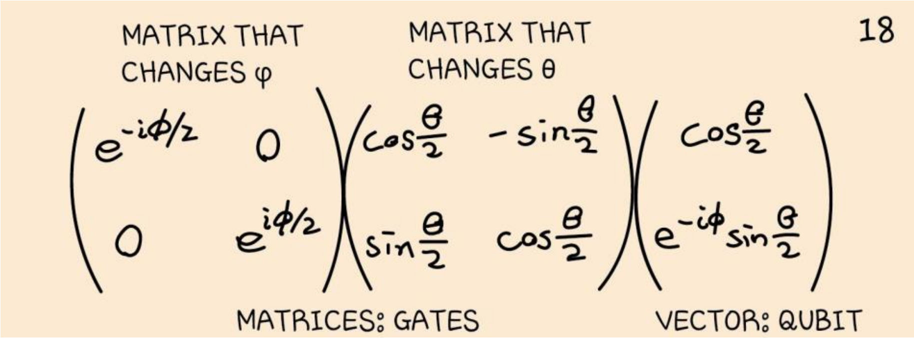
$$\nabla^2 H = \mu_0 \varepsilon_0 \frac{\partial^2 H}{\partial t^2}$$

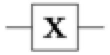
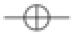
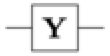
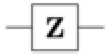
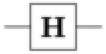
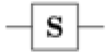
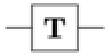
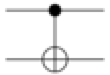
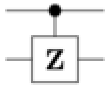
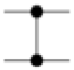
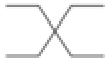
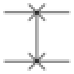
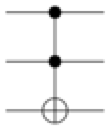
TELLS US THE SPEED

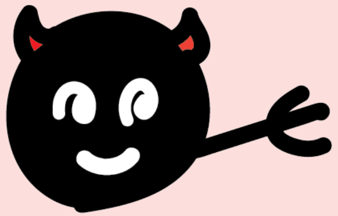
WAVE EQUATIONS

Gates (quantum operations)





Operator	Gate(s)		Matrix
Pauli-X (X)			$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Pauli-Y (Y)			$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
Pauli-Z (Z)			$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Hadamard (H)			$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
Phase (S, P)			$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
$\pi/8$ (T)			$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$
Controlled Not (CNOT, CX)			$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
Controlled Z (CZ)			$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$
SWAP			$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Toffoli (CCNOT, CCX, TOFF)			$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$



CONTROL QUBIT:
YOU STAY THE SAME IF I'M $|0\rangle$;
YOU CHANGE IF I'M $|1\rangle$.

TARGET QUBIT:
OKAY~

2020.4.20.

PRESERVE

$$\mathrm{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

SWITCH

A 4X4 MATRIX

The controlled-not gate manipulates the target qubit based on the state of the control qubit.

$\mathrm{CNOT}|00\rangle = |00\rangle$
$\mathrm{CNOT}|01\rangle = |01\rangle$
$\mathrm{CNOT}|10\rangle = |11\rangle$
$\mathrm{CNOT}|11\rangle = |10\rangle$

TRY THE MATH!

There are other controlled gates for multiple qubits you should look up.
We highlight CNOT as it will be used in every(?) algorithm (sounds familiar?!)

CNOT

$$CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Math insert - Matrix multiplication --

Gates are N by N matrices that multiply to state with 2^N vector elements. They follow the rules such that

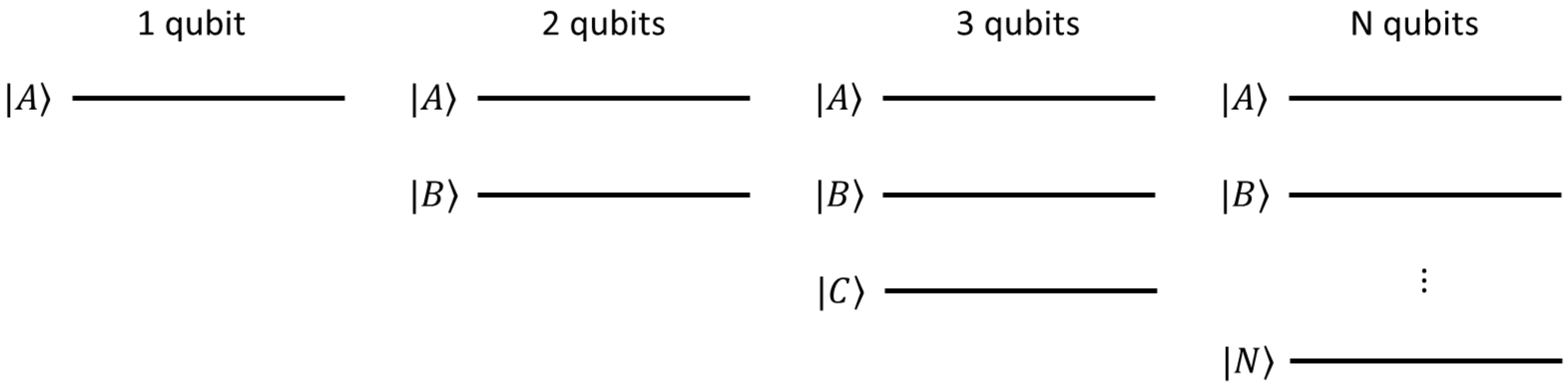
$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax+by \\ cx+dy \end{pmatrix},$$
$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} ax+by+cz \\ dx+ey+fz \\ gx+hy+iz \end{pmatrix},$$

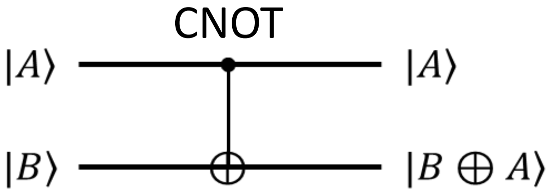
and so on.

$$CNOT|10\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = |11\rangle\,.$$

Similarly, $C|00\rangle = |00\rangle, C|01\rangle = |01\rangle$ and $C|11\rangle = |10\rangle$.

Circuit representation



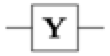
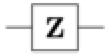
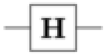
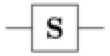
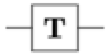
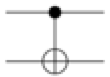
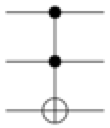


Target B controlled by A

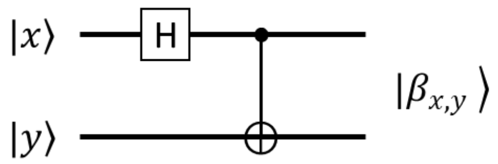
22

The Bloch sphere is no longer useful when we look at more than one qubit. But we have another graphic representation to use for multi-qubit systems.

Similar to how the lines in music scores denote the time-evolving music, we can use lines to represent the time-evolving qubit states:

Operator	Gate(s)	Matrix
Pauli-X (X)		$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Pauli-Y (Y)		$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
Pauli-Z (Z)		$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Hadamard (H)		$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
Phase (S, P)		$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
$\pi/8$ (T)		$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$
Controlled Not (CNOT, CX)		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
Controlled Z (CZ)		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$
SWAP		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Toffoli (CCNOT, CCX, TOFF)		$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

Creating Bell states (entanglement)



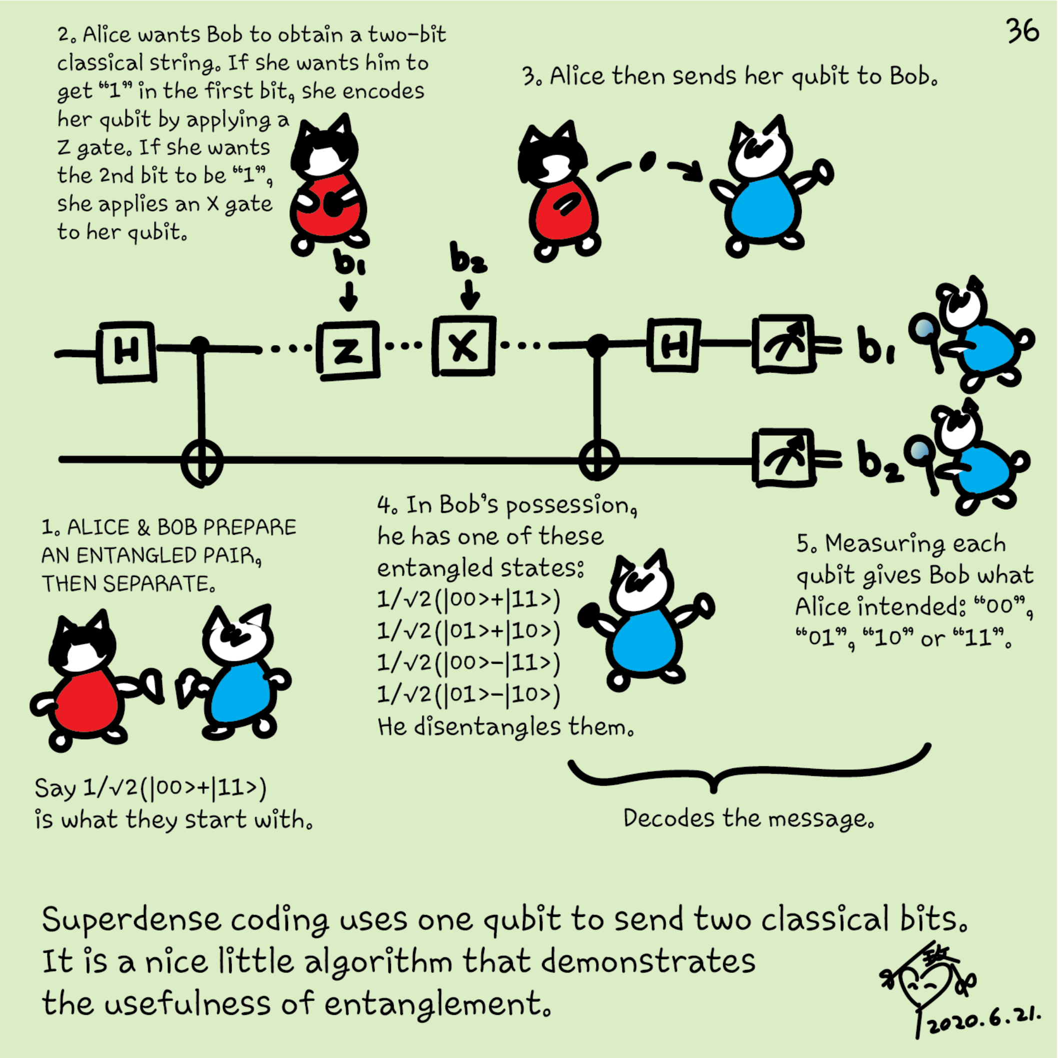
$H|10\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \otimes |0\rangle$

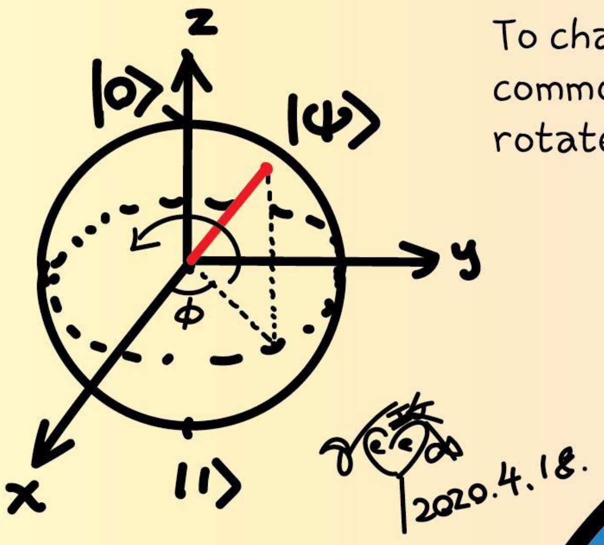
CNOT $\Rightarrow \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$

In	Out
$\|00\rangle$	$(\|00\rangle + \|11\rangle)/\sqrt{2} \equiv \|\beta_{00}\rangle$
$\|01\rangle$	$(\|01\rangle + \|10\rangle)/\sqrt{2} \equiv \|\beta_{01}\rangle$
$\|10\rangle$	$(\|00\rangle - \|11\rangle)/\sqrt{2} \equiv \|\beta_{10}\rangle$
$\|11\rangle$	$(\|01\rangle - \|10\rangle)/\sqrt{2} \equiv \|\beta_{11}\rangle$

Try proving this table

Superdense coding

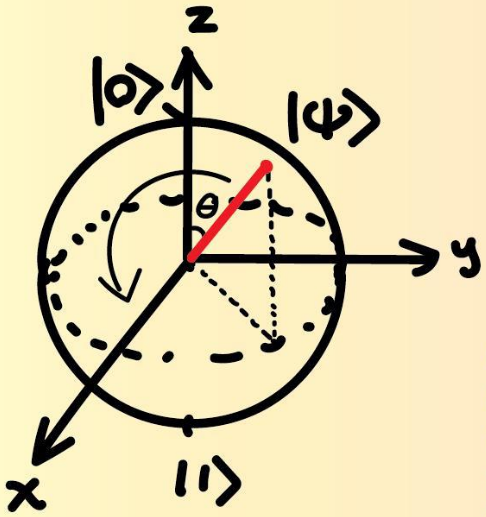




To change the phase φ, we have a commonly used gate, Z, which rotates about the z-axis by 180°.

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Similarly, the X gate rotates about the x-axis by 180°, rotating the angle θ e.g. $X|0\rangle = |1\rangle$, $X|1\rangle = |0\rangle$.



$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

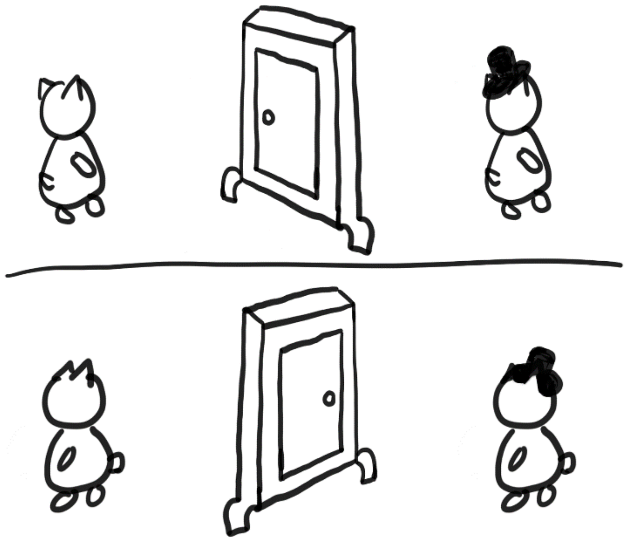
We have seen in page 18 the two matrices for changing φ and θ in arbitraty amounts. They form a universal gate set - they can put a state anywhere on the Bloch Sphere. The gates Z and X are special cases of them.

Q# exercise:

Option 1: No installation, web-based Jupyter Notebooks

- The Quantum Katas project (tutorials and exercises for learning quantum computing) https://github.com/Microsoft/QuantumKatas
- **SuperdenseCoding**
- Task 1.3 Adjoint, MResetZ
- https://docs.microsoft.com/en-us/qsharp/api/qsharp/microsoft.quantum.measurement.mresetz
- https://docs.microsoft.com/en-us/learn/modules/qsharp-create-first-quantum-development-kit/
- open Microsoft.Quantum.Measurement;

Gates



manipulate qubit states (vectors)
through matrix multiplications

unitarity $U^\dagger U = I$

So that it is reversible and probabilities add up to 1

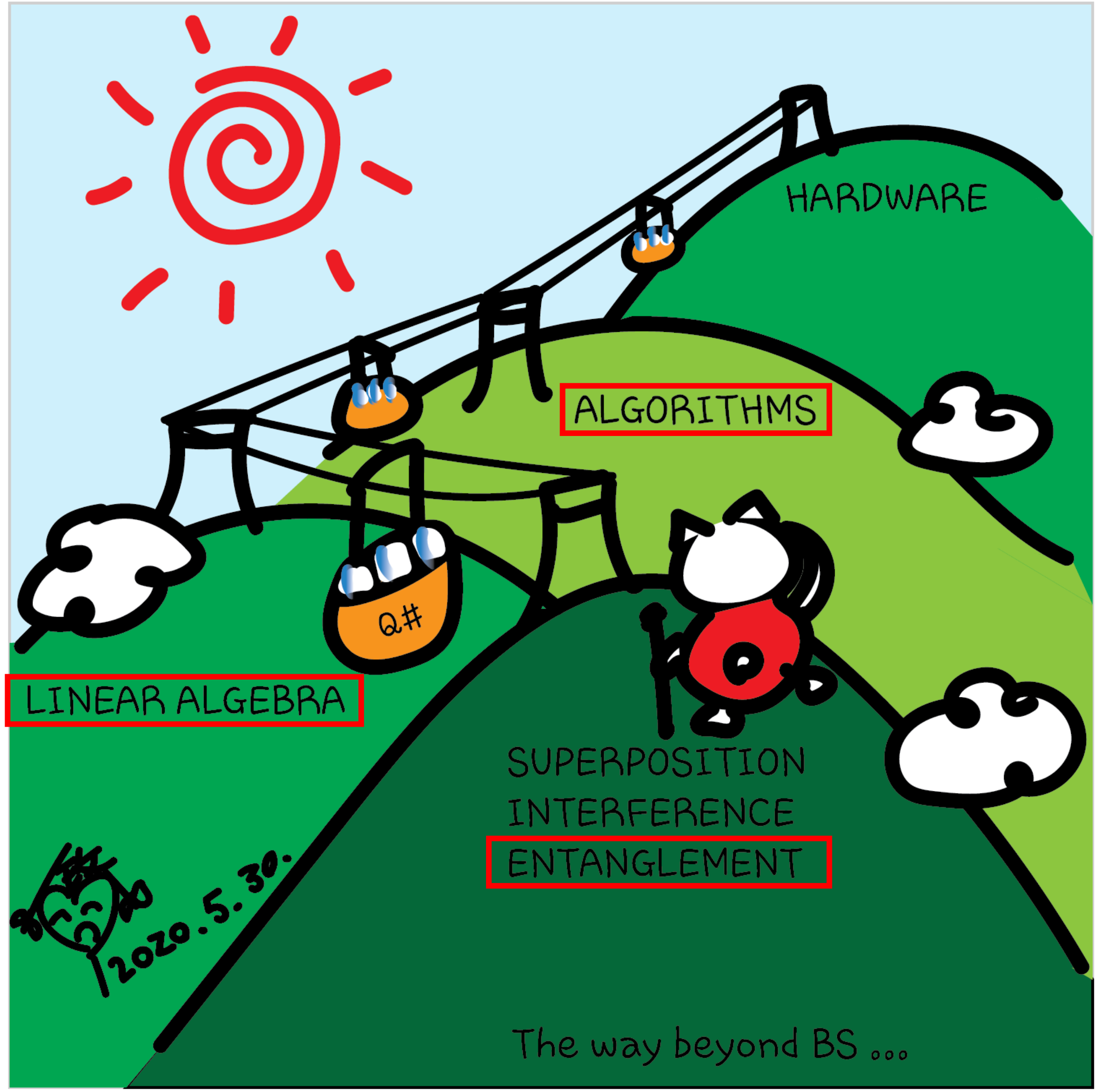
Math insert – unitary, adjoint or Hermitian conjugate --

In math, unitarity means $U^\dagger U = I$, where I is the identity matrix and the "†" symbol (reads "dagger") means adjoint or Hermitian conjugate of matrix U. It can be further written as $U^\dagger = (U^*)^T = (U^T)^*$, where "*T*" denotes transpose and "*" complex conjugate:

$$\begin{pmatrix} U_1 \\ U_2 \\ \vdots \\ U_N \end{pmatrix}^T = (U_1 \quad U_2 \quad \dots \quad U_N)$$

and if $a = a_0 + ia_1$, then $a^* = a_0 - ia_1$ by definition. Therefore,

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^\dagger = \begin{pmatrix} a^* & c^* \\ b^* & d^* \end{pmatrix}.$$



Superdense Coding
Teleportation
CHSH Game

For certificate 1

- Complete any one quantum kata
- Take a screenshot or photo
- Post on Twitter or LinkedIn
- Tag the following
- **Twitter:** @KittyArtPhysics @MSFTQuantum @QSharpCommunity #QSharp #QuantumComputing #comics #physics
- **LinkedIn:** @Kitty Y. M Yeung #MSFTQuantum #QSharp #QuantumComputing #comics #physics

Participate

- Microsoft Q# coding contest is happening from June 19 to June 22, 2020. Register now! https://codeforces.com/blog/entry/77614

- Azure Quantum Developer Workshop https://aka.ms/AQDW

Questions

- Post in chat or on Hackaday project https://hackaday.io/project/168554-introduction-to-quantum-computing

- Past Recordings on Hackaday project or my YouTube https://www.youtube.com/c/DrKittyYeung